Data-Driven Bending Elasticity Design by Shell Thickness

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Abstract
This technical report is a supplement of our work [ZLW] to design bending elasticity by shell thickness. General information and details can be found in [ZLW*]. In this report, comparison of data interpolation methods and algorithm of conventional ESN-based learning are described in details for further comprehension of the main paper.*

1. Comparison of Data Interpolation Learned by Different Methods

1.1. Learning by ESN

The function $F(\cdot)$ to be learned in our problem has its own characteristics, including highly nonlinear, low dimensional (i.e., two-dimensional using (D_i, M_i) as input) and small-sized(e.g., only 45 samples in our practice) . Moreover, the range of $F(\cdot)$ shoud be constrained. We employ *echo state network* (ESN) to learn the function, which results in the best quality of interpolation comparing to other methods. Prior works in computer graphics have conducted a variety of learning/data-interpolation tools such as *support vector machine* (SVM) [FVdPT01], *extreme learning machine* (ELM) [XXLX14, ZLP*15] and *radial basis functions* (RBF) [BBO*09]. The comparisons with them can be found in Fig.1. From Fig.1, we'll find because of the highly nonlinearity of our problem, SVM and ELM approaches are difficult to learn the function accurately, especially keeping the result $F(\cdot)$ in a reasonable range. Moreover, although RBF method is capable of accurate learning, overfitting problem is unavoidable. Besides the above neural networks methods, other machine learning methods are also investigated. For example, K-nearest neighbors algorithm is a common method for classification and regression. However, a shortcoming of the K-nearest neighbors algorithm is that it is sensitive to the local structure of the data. Random forest is also one of the prevailing learning methods but it is mainly used in classification and linear regression. To conclude, in order to satisfy our learning problem, we've applied echo state network method and modified the basic formulation to incorporate the range constraints.

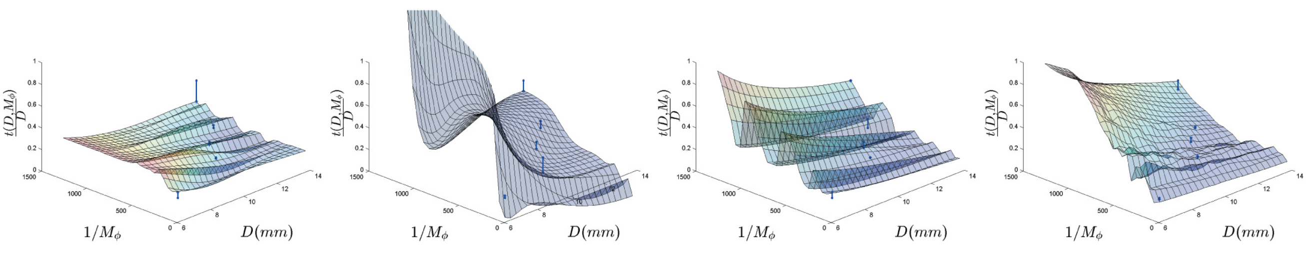

Figure 1: *Comparison of data interpolation on functions learned by different methods – from left to right, SVM (support vector regression approach with radius basis kernel), ELM (extreme learning machine for regression method with sigmoid activation functions), RBF (radius basis functions) and ESN (echo state network with range control), respectively. In all these learning methods, parameters have been tuned to obtain the best results by our training set. We then check the approximation errors on 5 new samples, the distances from which to the surface $t(D, M_\varphi)$ are illustrated by vertical line segments. Our ESN-based method results in smaller approximation errors to our knowledge.*

2. Conventional ESN-Based Learning

Assume an echo state network has n reservoir units, an k-dimensional input and a d-dimensional output, the neural network can be constructed and updated by the following four steps.

Step 1: Create a random dynamical reservoir neural network. The input signal $\mathbf{u}(z)$ is first attached to the reservoir units by creating random all-to-all connections $\mathbf{W}_{in}$. We then create random sparse connections in the reservoir. That is $\mathbf{W}$ – a sparse matrix with $n \times n$ dimensions. The neurons are linked to the output signal by an output weight matrix $\mathbf{W}_{out}$ to be determined via training. If the task requires output feedback, all-to-all connections linking output to reservoir are randomly generated as $\mathbf{W}_{fb}$. If no feedback is needed, we can assign a *zero* $\mathbf{W}_{fb}$. The state pf system in the reservoir is presented by $\mathbf{x}(z)$.

- The output can be obtained from $\mathbf{u}(z)$ and $\mathbf{x}(z)$ by Eq.(9) in [ZLW*].
- With the feedback from output signal, the system state is updated by Eq.(11) in [ZLW*].

Step 2: Harvest reservoir states. Given a training data set $\mathcal{D}$, the dynamical reservoir is driven dynamically for times $z = 1, 2, \ldots, m$. This results in a sequence of reservoir states, $\mathbf{x}(z)$, which is a nonlinear transform of the driving input.

Step 3: Compute output weights. The output weights are computed by the linear regression as shown in Eq.(10) in [ZLW*].

Step 4: Function evaluation. For a new input signal $\mathbf{u}(z+1)$, the new system state $\mathbf{x}(z+1)$ is first evaluated by Eq.(11) in [ZLW*] using the current system state $\mathbf{x}(z)$ and the last output signal $\mathbf{y}(z)$. The new output signal $\mathbf{y}(z+1)$ (i.e., the function value) is then computed by Eq.(9) in [ZLW*].

Appendix: Training Data

In our method, samples of tubes are measured to find the relationship between diameter, shell thickness and elasticity. Here 45 uniformly hollowed tubes with the same length (100mm in our tests) are first fabricated by 3D printing. Their diameters range from 6mm to 14mm and thicknesses range from 1mm to 6mm. Among these samples, 25 tubes were spaced uniformly across D and t, while the dimensions of the remaining 20 were randomly assigned. Moreover, 5 samples are generated for testing the interpolation error.

References

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Table 1: *Training Data and Testing Data*

Training Data			Testing Data		
Diameter (mm)	Thickness (mm)	Deflection (mm)	Diameter (mm)	Thickness (mm)	Deflection (mm)
6	1	40.01	6.4	1.16	29.72
6	1.25	35.28	7.38	2.16	15.22
6	1.5	33.4	8.4	3.9	9.79
6	1.75	31.81	8.8	2.64	8.23
6	2	31.04	10.4	1.5	5.8
8	1	14.89			
8	1.5	11.92			
8	2	11.06			
8	2.5	10.22			
8	3	10			
10	1	7.13			
10	1.75	5.26			
10	2.5	4.91			
10	3.25	4.18			
10	4	3.95			
12	1	4.08			
12	2	2.67			
12	3	2.18			
12	4	2.1			
12	5	2.06			
14	1	2.51			
14	2.25	1.72			
14	3.5	1.6			
14	4.75	1.55			
14	6	1.43			
6.96	1.72	18.39			
7.18	1.03	21.45			
7.96	1.24	14.87			
8.86	1.05	12.27			
9.2	1.9	8			
9.54	2.43	5.71			
9.6	2.97	5.56			
10	1.5	5.77			
10	2	5.21			
10.36	2.27	4.85			
10.9	3.1	3.36			
11.02	1.78	3.92			
11.28	2.6	2.93			
11.6	1.71	3.58			
12.3	2.48	3.01			
12.34	3.72	2.14			
12.58	1.02	3.61			
12.72	2.59	2.16			
12.88	1.52	2.48			
14	1.5	2.32			