Int J Adv Manuf Technol (2012) 62:1273–1280
DOI 10.1007/s00170-011-3847-8

ORIGINAL ARTICLE

An iterative feedrate optimization method for real-time NURBS interpolator

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Received: 28 August 2008 / Accepted: 9 December 2011 / Published online: 23 December 2011

Abstract Non-uniform rational B-spline (NURBS) interpolator has been widely used in modern manufacturing systems to machine arbitrary geometries with great relief of the data flow bottleneck and feedrate fluctuation. However, in practice, real-time feedrate does not always meet the computer numerical control (CNC) command exactly subject to the system dynamics. To solve this problem, we present a real-time NURBS interpolator with feedrate optimization for CNC machining tools in this work. The parametric curve is first approximated with the Adams–Bashforth method which provides uniform feedrate in each sampling period in the interpolation process. And then, a feedback scheme is introduced to adjust the feedrate in real time so as to guarantee a specified deviation between the measured and the desired feedrate. The convergence condition for this closed-loop algorithm is presented and analyzed. Simulation and real experiments on an X–Y table are employed to verify its feasibility. And the comparisons with traditional interpolators based on Taylor's expansion are also provided to demonstrate its improvement in accuracy.

Keywords NURBS · Feedrate optimization · Real-time control · Closed-loop control

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1 Introduction

Traditional computer-aided design (CAD) and computer-aided manufacturing (CAM) systems usually adopt linear and circular segments which jointed end-by-end as the modeling tools for free-form profiles. In order to meet the chord tolerance, complex profiles have to be divided into small segments, and consequently, the program code could swell to enormous size and data transmission load could be greatly increased [1–3]. That makes the feedrate fluctuation problem seriously and let the linear and circular interpolation methods be incapable to meet the requirements of high-speed and high-accuracy CNC machining tasks.

To overcome this drawback, a variety of parametric curves have been applied to parametric interpolation [4–9]. As a representative parametric means, NURBS has been widely used in CAD and CAM as it could provide a uniform representation for analytic shapes and free-form entities. NURBS has become an industry standard for profile representation, design, and data exchange of geometric information. Its particular mathematical properties allow a complex contour to be represented using only a few parameters [10]. Compared with the traditional linear or circular interpolation methods, the advantages of NURBS interpolator come from: (a) segmentation can be avoided, (b) data transmission load can be reduced, and (c) uniform speed can be achieved during the interpolation process. Moreover, NURBS technique has also been applied for chord error regularity [11], acceleration, and deceleration (ACC/DEC) planning [12], and representation of machine dynamic characters [13], etc. In this paper, we put our concern on the feedrate fluctuation problem in NURBS interpolator. The determination of feedrate in traditional

NURBS interpolators are usually open-loop based, and thus makes the feedrate estimation lack accuracy. To obtain a constant feedrate during the interpolation procedure and reduce the feedrate fluctuation, the parametric curve is approximated with the Adams–Bashforth method (ABM) [14] firstly. And then, a closed-loop algorithm is introduced to maintain the feedrate within a given tolerance range.

The rest of this paper is organized as follows: Related works of parametric interpolators are briefly reviewed in Section 2. The NURBS curve and traditional interpolators are introduced in Section 3. The proposed closed-loop feedrate optimization scheme and its convergence condition are presented and analyzed in Section 4. Experimental results can be found in Section 5. Conclusion and future works are offered in Section 6.

2 Related works

Major concerns among different parametric interpolators mainly focused on following aspects: feedrate, chord error, ACC/DEC, and jerk. Machining errors are usually caused by the feedrate fluctuation, inappropriate ACC/DEC planning, system dynamics, and sharp corners, etc. In order to reduce the feedrate fluctuation, constant parametric increment is used in the uniform interpolation algorithm [15]. In [16], a parametric curve interpolator with its corresponded segmentation that based on uniform curve length rather than uniform parametric increment is introduced. To calculate the location of the new sampling point, first order Taylor's expansion method is used to achieve a constant feedrate. A continuous work is presented in [4], which improves the result by extending Taylor series coefficients into second order. To improve the feedrate estimation accuracy, a first-order approximation algorithm with a compensatory value is introduced in [17]. Then, the proposed interpolator is applied to the constant-speed mode and the ACC/DEC mode to achieve constant feedrate. In [18], high-order terms in the traditional Taylor's expansion method are neglected in calculation of new sampling point. And a feedback loop is presented to reduce the feedrate fluctuation caused by the truncation error.

To obtain a confined chord error, an adaptive feedrate interpolation algorithm is presented in [11]. Since the chord error is related to both feedrate and the radius of curvature, the feedrate has to be changed adaptively with respect to the curvature during the interpolation process. A variable feedrate interpolator is proposed to apply the ACC/DEC planning before feedrate interpolation [12]. And the results showed that the second-order variable feedrate interpolator with bell-shape ACC/DEC planning function can outperform. In [19], a method that could detect the sharp corners is proposed by the use of a look-ahead function. Although the authors declare that the feedrate can be adjusted adaptively in real time, but no experimental results are provided. In [20], the sharp corners are predetermined in an off-line mode. The proposed interpolator can confine the chord error in a specified tolerance and control feedrate and ACC/DEC of machining during the interpolation process. In [21], a two-stage method is introduced to decrease the interpolation error. The feedrate is first lowered to the maximum allowable feedrate that satisfies the interpolation error condition. And then, the kinematical properties generated in the first stage are improved by using curve look-ahead and acceleration characteristic equations. A new method that involved chord error, feedrate, acceleration, and jerk limitations are presented in [22]. Sampling points are determined at a look-ahead stage by previewing the curve. Several models are presented to decide the minimum feedrate in terms of different kinematic conditions. And then, a real-time stage is implemented to determine a jerk-limited kinematic profile. However, a unique representation for different kinematic conditions is not provided. A synthesized coordination control algorithm for a bi-axial tracking control system is introduced in [13]. The disturbance attenuation scheme is first implemented to reduce the influence caused by friction and load disturbance. And then an enhancement scheme is employed to reduce contour error and improve both tracking and contouring performances. However, as pointed out by the authors, the parameters of machine dynamic characters should be known as a priori.

3 NURBS interpolator

The explicit function for parametric curve in 3-dimensional space can be generalized as:

$$C(u) = (x(u), y(u), z(u)) \quad a \leq u \leq b \tag{1}$$

where u indicates an arbitrary independent parameter which is usually normalized to [0,1]. A p-th degree NURBS curve [10] can be defined as:

$$C(u) = \sum_{i=0}^{n} N_{i,p}(u) w_i P_i \Big/ \sum_{i=0}^{n} N_{i,p}(u) w_i \quad a \leq u \leq b \tag{2}$$

where $\{P_i\}$ refers to the control points, $\{w_i\}$ are the weights, and $\{N_{i,p}(u)\}$ are the p-th degree B-spline basis functions which are defined on the non-periodic and non-uniform knot vector U:

$$U = \{\underbrace{a, \ldots, a}_{p+1}, u_{p+1}, \ldots, u_{m-p-1}, \underbrace{b, \ldots, b}_{p+1}\}$$

$N_{i,p}(u)$ can be defined by the following recursive formulations:

$$N_{i,0}(u) = \begin{cases} 1 & u_i \le u < u_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad N_{i,p}(u) = \frac{u-u_i}{u_{i+p}-u_i} N_{i,p-1}(u) + \frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}} N_{i+1,p-1}(u) \tag{3}$$

The *m*-th derivative of the NURBS with respect to *u* can be obtained by:

$$C^{(m)}(u) = \sum_{i=0}^{n} N_{i,p}^{(m)}(u) P_i \tag{4}$$

where the *m*-th derivative of $N_{i,p}$ (u) can be expressed as:

$$N_{i,p}^{(m)}(u) = p\left(\frac{N_{i,p-1}^{(m-1)}(u)}{u_{i+p} - u_i} - \frac{N_{i+1,p-1}^{(m-1)}(u)}{u_{i+p+1} - u_{i+1}}\right)$$

To reduce the feedrate fluctuation, sampling point is selected based on uniform curve length ΔS_i [16]. As a result, for a fixed sampling period *T*, feedrate during each period keeps constant. Feedrate along NURBS curve can be defined as:

$$V(t) = \frac{\mathrm{d}s}{\mathrm{d}t} = \left(\frac{\mathrm{d}s}{\mathrm{d}u}\right)\left(\frac{\mathrm{d}u}{\mathrm{d}t}\right) \quad \text{or} \quad \frac{\mathrm{d}u}{\mathrm{d}t} = \frac{V(t)}{\mathrm{d}s/\mathrm{d}u} \tag{5}$$

where, $\frac{ds}{du} = \sqrt{(x')^2 + (y')^2 + (z')^2}$, and $x' = \frac{dC_x(u)}{du}, y' = \frac{dC_y(u)}{du}, z' = \frac{dC_z(u)}{du}$. Equation 5 can be rewritten as:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{V(t)}{\sqrt{(x')^2 + (y')^2 + (z')^2}} \tag{6}$$

It is difficult to obtain the closed-form solution for Eq. 6, since the representation of $C(u)$ is an infinite sum. Therefore, an alternative recursive solution based on Taylor's expansion method around $t=kT$ can be adopted as:

$$u_{k+1} = u_k + T\frac{\mathrm{d}u}{\mathrm{d}t}\bigg|_{t=kT} + \frac{T^2}{2}\frac{\mathrm{d}^2u}{\mathrm{d}t^2}\bigg|_{t=kT} + \varepsilon_{u=u_k} \tag{7}$$

The second-order derivative of *u* with respect to *t* can be denoted as:

$$\frac{\mathrm{d}^2u}{\mathrm{d}t^2} = -\frac{V(t)\cdot(x'\cdot x'' + y'\cdot y'' + z'\cdot z'')\cdot\frac{\mathrm{d}u}{\mathrm{d}t}}{(x'^2 + y'^2 + z'^2)^{\frac{3}{2}}} \tag{8}$$

where $x'' = \frac{d^2C_x(u)}{du^2}, y'' = \frac{d^2C_y(u)}{du^2}, z'' = \frac{d^2C_z(u)}{du^2}$

By substituting Eqs. 6 and 8 into Eq. 7, the equation of 3-dimensional parametric curve interpolator based on the second-order Taylor's expansion can be normalized to Eq. 9 as:

$$u_{k+1} = u_k + \frac{TV(t)}{\sqrt{(x'^2 + y'^2 + z'^2)}}\bigg|_{t=KT} - \frac{T^2V^2(t)\cdot(x'\cdot x'' + y'\cdot y'' + z'\cdot z'')}{2\cdot(x'^2 + y'^2 + z'^2)^2}\bigg|_{t=KT} \tag{9}$$

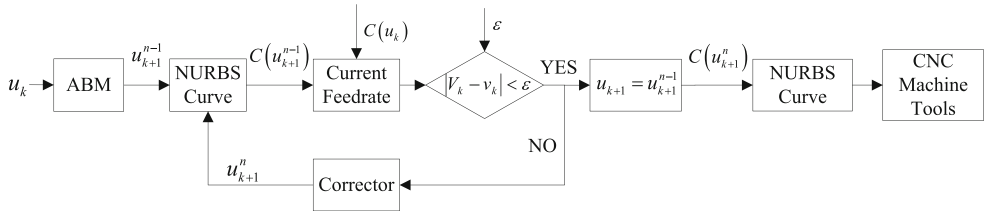
As shown in Eq. 9, the value of t_{k+1} sampling point $C(u_{k+1})$ can be calculated by u_k and the first- and second-order derivative of current position $C(u_k)$ with respect to the parameter *u*. By repeating the above process, a vector of *u* can be obtained to determine a parametric curve with its segmentation based on uniform curve length.

4 ABM-based iterative feedrate optimization

Traditional NURBS interpolators are usually with open-loop style. And that makes it difficult to maintain a constant feedrate in manufacturing process [12]. To overcome this limitation, in this paper, a closed-loop feedrate optimization method is proposed to calculate u_{k+1} so as to guarantee the feedrate fluctuation within a tolerance range. Flowchart of the proposed scheme is shown in Fig. 1. The systematic process can be described as follows:

1. u_k is used to estimate u_{k+1} initially by applying the Adams–Bashforth method;
2. The next sampling point $C(u_{k+1})$ is determined by the NURBS curve presentation;
3. The current feedrate is calculated from the adjacent sampling points;
4. Judge if the deviation between current and desired feedrate satisfies a pre-defined feedrate tolerance;
5. If YES, u_{k+1} is appointed as the next parameter and $C(u_{k+1})$ are the next sampling point. Then, the machining command is sent to CNC machine;
6. If NO, the corrector is started to adjust u_{k+1}. Repeating steps 1 to 4 until an acceptable feedrate tolerance is achieved.

Fig. 1 Workflow of the proposed NURBS interpolator with feedrate optimization



4.1 Feedrate optimization strategy

For simplification, Eq. 6 can be rewritten as:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{V(t)}{\mathrm{d}s/\mathrm{d}u} = f(u) \tag{10}$$

The Adams–Bashforth method is adopted to solve Eq. 10, and the initial estimation of u_{k+1} can be expressed as:

$$u_{k+1} = u_k + \frac{T}{24}(-9f(u_{k-3}) + 37f(u_{k-2}) - 59f(u_{k-1}) + 55f(u_k)) \tag{11}$$

where T indicates the sampling period.

For the parameters, u_0 is initialized to 0, u_1, u_2, u_3 are determined by the fourth-order Runge–Kutta method [14] as given by the following equation:

$$u_{k+1} = u_k + \frac{T}{6}(f_{k1} + 2f_{k2} + 2f_{k3} + f_{k4}) \tag{12}$$

where

$$\begin{aligned} f_{k1} &= f(t_k, u_k) \\ f_{k2} &= f(t_k + \tfrac{T}{2}, u_k + \tfrac{Tf_{k1}}{2}) \\ f_{k3} &= f(t_k + \tfrac{T}{2}, u_k + \tfrac{Tf_{k2}}{2}) \\ f_{k4} &= f(t_k + T, u_k + Tf_{k3}) \end{aligned}$$

The following equation is used to obtain a corrected estimate of u_{k+1}:

$$u_{k+1}^{n} = u_k + \frac{V_k}{v_k^{n-1}}(u_{k+1}^{n-1} - u_k) \tag{13}$$

where V_k is the desired feedrate at time t_k, and v_k^{n-1} is the current feedrate after n-1 iterations, which is defined as:

$$v_k^{n-1} = \frac{|C(u_{k+1}^{n-1}) - C(u_k)|}{T} \tag{14}$$

Let $\varepsilon^{n-1} = |v_k^{n-1} - V_k|$ be the feedrate error after n-1 iterations and ε be the pre-defined feedrate tolerance, the termination condition for the iteration is:

$$\varepsilon^{n-1} \leq \varepsilon \tag{15}$$

To end the iteration, the termination condition must be met after several times of iteration. In this case, the sampling period T must be satisfied with the convergence condition, and it will be presented in the following section.

4.2 Analysis of convergence condition

Convergence of functional iteration requires a tiny step size T. However, smaller step size increases iteration times and decreases the computing efficiency. In this section, the convergence condition of the iterative method is analyzed and discussed.

By subtracting u_{k+1}^{n-1} from u_{k+1}^{n}, we can get:

$$u_{k+1}^{n} - u_{k+1}^{n-1} = \frac{V_k}{v_k^{n-1}}(u_{k+1}^{n-1} - u_k) - \frac{V_k}{v_k^{n-2}}(u_{k+1}^{n-2} - u_k) \tag{16}$$

where

$$\begin{aligned} u_{k+1}^{n} &= u_k + \tfrac{V_k}{v_k^{n-1}}(u_{k+1}^{n-1} - u_k) \\ u_{k+1}^{n-1} &= u_k + \tfrac{V_k}{v_k^{n-2}}(u_{k+1}^{n-2} - u_k) \\ v_k^{n-1} &= \tfrac{|C(u_{k+1}^{n-1}) - C(u_k)|}{T} \\ v_k^{n-2} &= \tfrac{|C(u_{k+1}^{n-2}) - C(u_k)|}{T} \end{aligned}$$

Equation 16 can be rewritten as:

$$u_{k+1}^{n} - u_{k+1}^{n-1} = \frac{TV_k}{|C(u_{k+1}^{n-1}) - C(u_k)|}(u_{k+1}^{n-1} - u_k) - \frac{TV_k}{|C(u_{k+1}^{n-2}) - C(u_k)|}(u_{k+1}^{n-2} - u_k) \tag{17}$$

Denote h=u_{k+1}-u_k, the derivative of $C(u)$ with respect to u at u_{k+1} can be defined as:

$$C'(u_{k+1}) = \lim_{h \to 0} \frac{C(u_{k+1}) - C(u_k)}{h} \tag{18}$$

Equation 17 can be rewritten as:

$$u_{k+1}^{n} - u_{k+1}^{n-1} = TV_k \left(\frac{1}{|C'(u_{k+1}^{n-1})|} - \frac{1}{|C'(u_{k+1}^{n-2})|} \right) \tag{19}$$

Since $\frac{1}{|C(u)|}$ is everywhere differentiable and the absolute value of its derivation is bounded, the Lipschitz condition [23] can be written as:

$$\left| \frac{1}{|C'(u_{k+1}^{n-1})|} - \frac{1}{|C'(u_{k+1}^{n-2})|} \right| \leq L_1 |u_{k+1}^{n-1} - u_{k+1}^{n-2}| \tag{20}$$

where L_1 indicates the Lipschitz constant which is defined as the maximum value of $\frac{d}{du}\frac{1}{|C'(u)|}$ with $u \in [u_{k+1}{}^{(n-2)}, u_{k+1}{}^{(n-1)}]$:

$$L_1 = \max\left|\frac{d}{du}\frac{1}{|C'(u)|}\right| \quad (21)$$

By substituting Eq. 20 into Eq. 19, we have:

$$|u_{k+1}^n - u_{k+1}^{n-1}| \le TV_k L_1 |u_{k+1}^{n-1} - u_{k+1}^{n-2}| \quad (22)$$

By repeating the above process, following equation can be obtained:

$$|u_{k+1}^n - u_{k+1}^{n-1}| \le TV_k L_1 \cdot TV_k L_2 \cdot \ldots \cdot TV_k L_{n-1} |u_{k+1}^1 - u_{k+1}^0| \quad (23)$$

Let L denotes the maximum value of L_i (i=1,2,…,n-1), Eq. 23 can be expressed as:

$$|u_{k+1}^n - u_{k+1}^{n-1}| \le (TV_k L)^{n-1} |u_{k+1}^1 - u_{k+1}^0| \quad (24)$$

Since u_{k+1}^1 and u_{k+1}^0 are bounded, the sequence u_{k+1}^n will be convergent when TV_kL is under the convergence condition: TV_kL <1.

Assume L occurs after n-iteration: $\left|\frac{d}{du}\frac{1}{|C'(u)|}\right|_{u=u_{k+1}^n}$, and the convergence condition is $TV_k\left|\frac{d}{du}\frac{1}{|C'(u)|}\right|_{u=u_{k+1}^n} < 1$, we can deduce:

$$T < 1/V_k\left|\frac{d}{du}\frac{1}{|C'(u)|}\right|_{u=u_{k+1}^n} \quad (25)$$

Hence, Eq. 25 is the convergence condition for the proposed interpolator. And u_{k+1}^n, u_{k+1}^{n-1} approximate to 0 as n approaches to ∞. From Eq. 13, the current feedrate v_k^{n-1} could approach the desired feedrate V_k eventually.

5 Experimental results

The experiments are conducted on both simulation data and an X–Y table. The results are evaluated via accuracy and efficiency in comparison with traditional NURBS interpolators. Figure 2 shows a NURBS tool path with its control points, weight vector, and knot vector are given as:

Control points:

{(0, 0), (−100, −100), (−100, 100), (0, 0), (100, −100), (100, 100), (0, 0)};

Weight vector:

{5, 5, 10, 1, 10, 5, 5};

Knot vector:

{0, 0, 0, 0.25, 0.5, 0.5, 0.75, 1, 1, 1}.

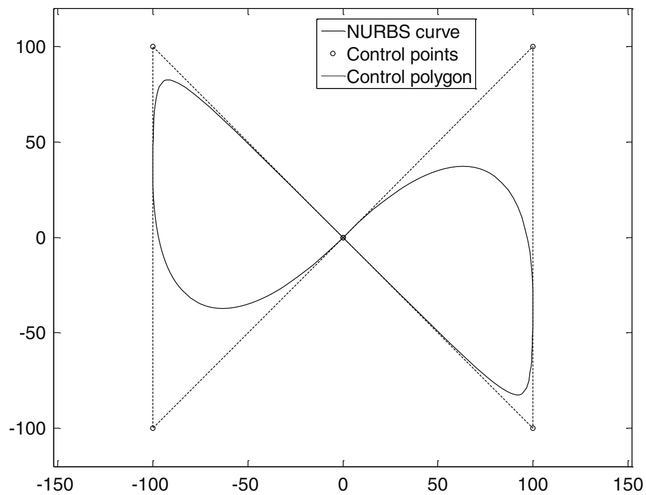


Fig. 2 Contour profile represented by NURBS curve

The length of the tool path is S=679.523 mm, which is calculated by Simpson's rule [23] and the sampling period is set to T=0.008 s according to Eq. 25. Other parameters of the interpolator are set as follows:

The maximum feedrate: V=100 mm/s;
Acceleration: A=150 mm/s^2;
Feedrate tolerance: ε=3 mm/s.

5.1 Experiment with simulation data

With above parameters, total interpolation time is calculated to 7.462 s. Table 1 shows the simulation results by NURBS interpolators with first- and second-order Taylor's expansion method and the result by the proposed method. From the results, we can see that with the proposed method, the feedrate fluctuation ranges from −1.351 to 1.720 mm/s, which can be controlled within the pre-defined feedrate tolerance and much smaller than that obtained by the Taylor's expansion methods.

The feedrate profiles and feedrate errors are plotted in Fig. 3. From the results, we can see that the proposed method can outperform distinctly than traditional NURBS interpolators. Constant feedrate with a much smaller feedrate error can be achieved via the proposed feedrate optimization method.

Table 1 Simulation results by three interpolators (millimeters per second)

	1st-order Taylor's expansion	2nd-order Taylor's expansion	Our method
Feedrate error peek	5.530	3.475	1.720
Feedrate error valley	−6.379	−4.557	−1.351

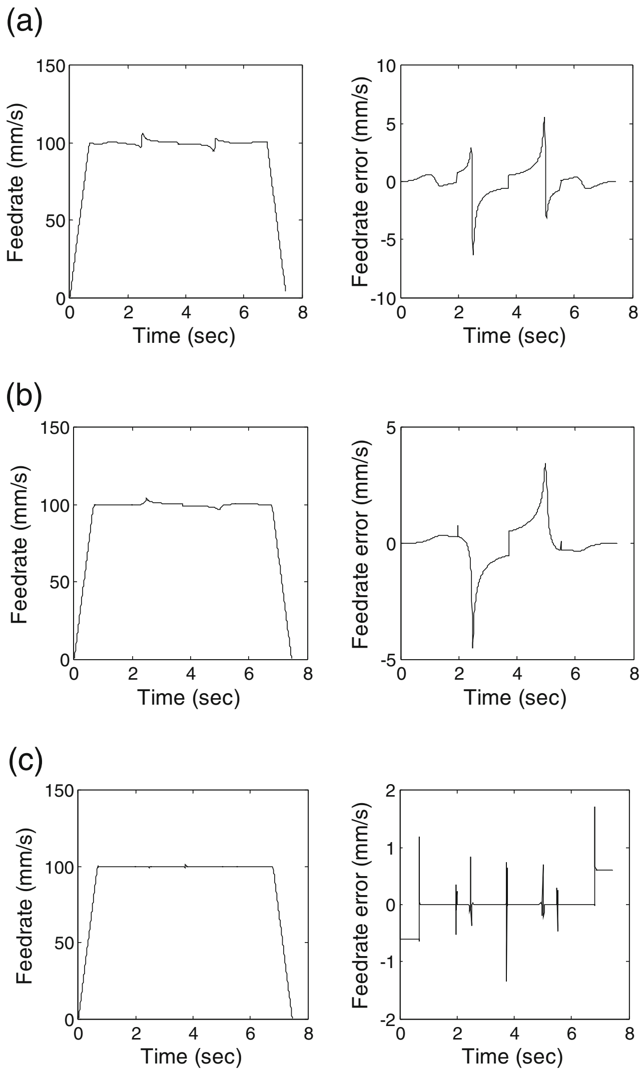


Fig. 3 Feedrate profiles and errors via simulation test. **a** Results by first-order Taylor's expansion method; **b** results by second-order Taylor's expansion method; **c** results by the proposed method

5.2 Real experiment with an X–Y table

An experimental system is implemented to verify the real-time performance of the proposed interpolator. Figure 4 shows the experimental setup which is developed on an X–Y table controlled by a motion control board with a TI TMS320F2812 DSP. Grating measurements are used to detect transient position of the X–Y table. Transient feedrate on X and Y directions can be calculated respectively by the transient position. Note that the pitch of screw is 16 mm in each direction. The X–Y table is driven by two DC servo motors and each motor performs a resolution of 4,000 steps per revolution.

By the proposed algorithm, the number of interpolation steps is around 1,400 and the average step length is about 5×10^{-3} mm. The contour error and feedrate error is used to evaluate the performance of the algorithms. As shown in Fig. 5, the obtained contour by our algorithm can fit the desired contour very closely. Some cropped areas are provided for close observation. Interpolation methods using the first- and second-order Taylor's expansion algorithm are also implemented to provide comparative results as shown in Table 2. By using the first and second order of Taylor's expansion methods, the maximum contour error is 0.710 and 0.465 mm, respectively. In contrast, our proposed method performs 0.165 mm, which is the lowest among them. Specifically, the maximum feedrate error reaches 7.752 and 6.343 mm/s for the first and second order of Taylor's expansion method, respectively. In comparison, feedrate of our proposed method maintains relatively stable, and the maximum feedrate error is only about 3.212 mm/s.

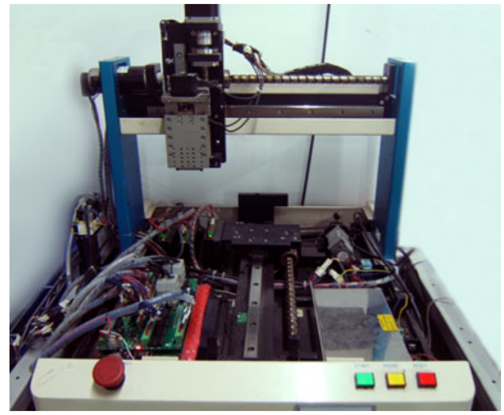


Fig. 4 The X–Y table used for real experiment

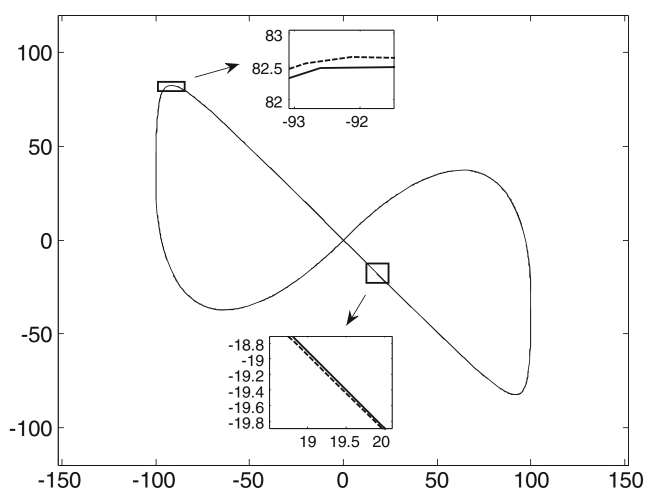


Fig. 5 Desired contour (*dashed line*) and resulting contour (*solid lines*) with the proposed method

Table 2 Experimental results for three interpolators

	1st-order Taylor's expansion	2nd-order Taylor's expansion	Our method
Maximum contour error(mm)	0.710	0.465	0.165
Maximum feedrate error(mm/s)	7.752	6.343	3.212

Contour error and feedrate profile by the three interpolators are plotted in Fig. 6. From the results, we can see that the contour error remains at a much lower range by the proposed method than that by the first and second order of Taylor's expansion methods. Moreover, by the proposed method, feedrate fluctuation can be distinctly decreased in comparison with the Taylor's expansion methods.

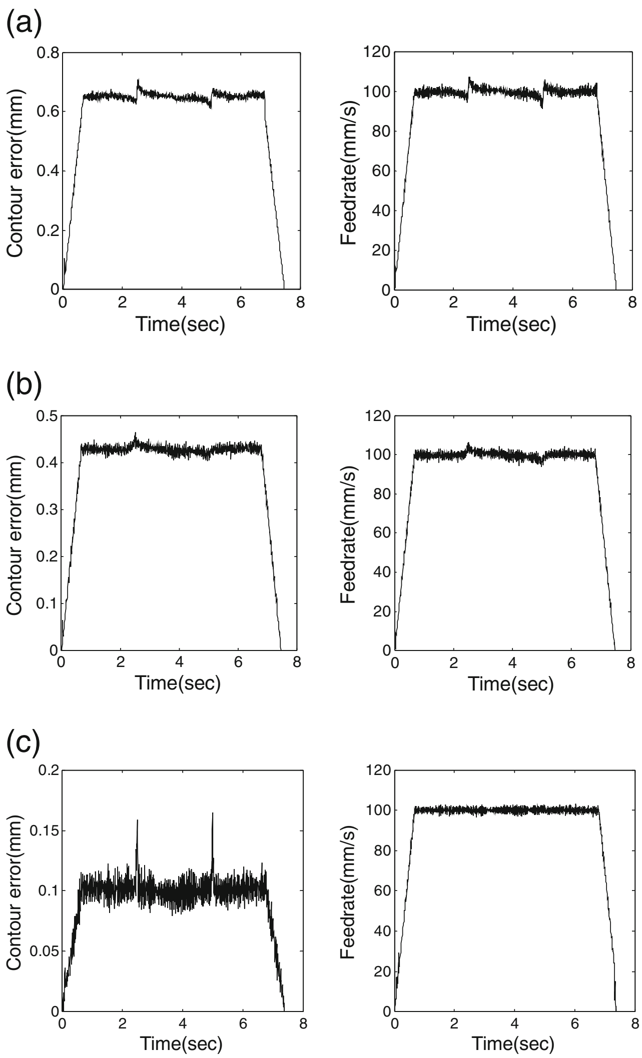


Fig. 6 Contour error and feedrate profile by the methods of **a** first-order Taylor's expansion, **b** second-order Taylor's expansion, and **c** the proposed method

6 Conclusion and future work

In this paper, a novel feedrate optimization algorithm for real-time NURBS interpolation is proposed. The Adams–Bashforth method (ABM) is used to calculate the initial position of the target sampling point, and current feedrate can be obtained with reference to the adjacent sampling points. A closed-loop optimization scheme is introduced to adjust the parameters of the target sampling point so as to keep the deviation between current and desired feedrate within a pre-defined tolerance. The convergence condition for the closed-loop feedrate optimization algorithm is also presented and analyzed. Simulation and real experiments are conducted to show that our proposed method could achieve higher accuracy than the traditional NURBS interpolators such as the first- and second-order Taylor's expansion methods. Future work can address how to integrate other error decreasing schemes such as chord error reduction, ACC/DEC planning, and system dynamic characters analysis into the proposed optimization method so as to further improve its performance.

Acknowledgments The work described in this article was supported partially by the grants from the Introduced Innovative R&D Team of Guangdong Province “Robot and Intelligent Information Technology”, Shenzhen Key Laboratory of Precision Engineering (project no. CXB201005250018A), National Natural Science Foundation of China (NSFC, grant no. 61002040), and NSFC-GuangDong (grant no. 10171782619-2000007).

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